J. Fluid Mech. (1996), *vol.* 314, *pp.* 349–371

Internal waves, turbulence and mixing in stratified flows: a report on Euromech Colloquium 339

By C. STAQUET AND J. SOMMERIA

Laboratoire de physique, Ecole Normale Supérieure de Lyon, 46 allée d'Italie, 69364 Lyon Cédex 07, France

(Received 4 December 1995)

Euromech colloquium 339 was organized by C. Staquet in Lyon (France) from September 6 to 9, 1995. It involved seventy-six participants from fourteen countries. Papers were presented on various aspects of stably stratified flows: (i) internal waves, their generation mechanisms, propagation and reflection properties, their instabilities leading to breaking; (ii) vortex structures in stably stratified fluids, which can be slow layerwise structures, or small intense vortices, appearing for instance in shear flow instabilities; (iii) statistical properties of random wave fields or stratified turbulence; (iv) mixing properties resulting from internal wave fields and stratified turbulence. These quite different dynamical regimes are often closely connected in actual flow problems, and one purpose of this colloquium was to better understand these connections. Participants were interested in fundamental aspects or in more specific applications, in engineering, geophysics and astrophysics. The colloquium was a rare opportunity to gather together scientists with these different points of view, to compare approaches and results, and to highlight general problems.

1. Introduction

Internal (gravity) waves propagate in any stably stratified fluid medium due to the restoring effect of the buoyancy force. As detailed by Etling*, internal waves at scales from a few metres to a few kilometres are observed in various geophysical flows. They are revealed by tracers such as clouds in the atmosphere, by direct measurements (of velocity, pressure, potential temperature in the atmosphere and of density in oceans or lakes), and by indirect measurements of the remote sensing type (such as RADAR). The SAR (Synthetic Aperture Radar), in being sensitive to the roughness of the sea surface, is able to show 'footprints' on that surface of oceanic internal waves. These waves have a significant role in the overall dynamics, mainly through their associated momentum and energy fluxes. Internal waves also arise in quite a variety of media, such as stellar interiors or industrial devices (see conclusions in §6).

In water, the stratification is generally due to temperature or salinity gradients. The density is often considered as a linear function of the temperature or salinity, transported by advection and diffusion. The buoyancy is then just proportional to the transported quantity. However, water density is generally a nonlinear function of temperature (and compressibility may be significant in deep water). Zimmerman*

* Indicates a speaker at the colloquium.

also pointed out that difficulties arise in an ethylene–glycol mixture or other non-ideal solutions, and that non-equilibrium thermodynamics must be introduced. Compressibility has to be taken into account in the atmosphere, and potential temperature is then the transported quantity.

Internal waves propagating along a density interface behave similarly to surface waves. Different kinds of internal solitary waves can be obtained for instance in shallow water conditions, as discussed by Zimmerman* (Zimmerman & Velarde 1994). The result of wave breaking is, however, different from surface waves, as it involves irreversible mixing effects at the interface.

In a continuously stratified fluid, internal waves are fundamentally different to surface waves because they propagate in an anisotropic medium. The restoring buoyancy force acting on a vertically displaced fluid particle is characterized by the Brunt-Väisälä (or buoyancy) frequency: $N(z)^2 = -(g/\rho)\partial\rho/\partial z$, in an incompressible fluid; when compressibility effects are important, the vertical gradient $\partial\rho/\partial z$ of the density profile $\rho(z)$ must be replaced by its difference from the adiabatic gradient. In a uniformly stratified fluid, i.e. with constant N, the well-known dispersion relation of internal waves: $\omega = Ncos(\theta)$, is obtained, ω being the intrinsic frequency of the wave and θ the angle of its wavevector with the horizontal. The wave frequency ω does not depend upon the magnitude of the wavevector, so that the group velocity, which carries the wave energy, is perpendicular to the wavevector, while the phase velocity is along it (see e.g. Lighthill 1978).

Weak internal waves in a uniformly stratified fluid interact by triad resonant interactions, obtained at the second order in the expansion in wave amplitude. By contrast surface waves interact only at higher order in the expansion (see Phillips 1981 for a review). Some of these resonant interactions slowly transfer energy to higher wavenumbers (and lower frequency), resulting in higher nonlinearity, so that breaking eventually occurs. Internal waves also break and produce sporadic turbulence under various conditions, discussed by several speakers. Shear instability or local unstable overturns then occur, and generate small intense vortices. Inversely, strong turbulence is a source of internal waves. Slow pancake vortices often coexist with internal waves and the interactions of these two kinds of motion is still poorly understood.

Quite different dynamical regimes therefore interact, and a colloquium on internal waves could not ignore strongly nonlinear effects, turbulence and induced mixing. The colloquium was thus composed of five sessions, starting with linear internal waves, and their mechanisms of instability and breaking. The deterministic aspects discussed in these two sessions are reported in §2. The third session was devoted to the various vortex structures, and corresponds to §3. The two last sessions were about the statistical properties of stratified turbulence and about mixing. These two subjects are of course closely related since mixing is due to the random displacement of fluid particles. They are reported in §§4 and 5.

Finally, it is useful to recall the main non-dimensional parameters that characterize a stably stratified flow. When there is no mean shear, buoyancy and inertial effects are compared through a Froude number $Fr = U/NL$, where U and L are a typical velocity and length scale of the flow. Internal waves correspond to motions with small Fr, and this parameter is then related to the slope of the deformed isodensity lines. These lines locally overturn when $Fr \sim 1$, leading to a (statically) unstable stratification. Such overturning generally produces strong turbulence by convective (or Rayleigh–Taylor) instability, although the details are more complex (see §2). When $Fr \gg 1$ the buoyancy is transported without significant effect on the flow: it behaves like a passive tracer. When a mean shear is present, a Richardson number Ri is generally preferred, which quantifies the relative strength of the density (or temperature) gradient and of the shear. If we denote the shear as U/L, the Richardson number is in fact $1/Fr^2$.

2. Internal waves: from build-up to breaking

We first present results about internal wave emission, either from a spherical oscillating source, a topography or a flow. Papers dealing with reflection properties of an internal wave field are presented next. We finally report on different mechanisms of instability and breaking of an internal wave field.

2.1. *Emission of internal waves*

The build-up of an internal wave field emitted by an oscillating source was addressed by Voisin*. The fluid is uniformly stratified. The emission of internal waves by a two-dimensional monochromatic oscillating source of finite extent in a viscous fluid has been calculated by Thomas & Stevenson (1972) (see Lighthill 1978 for a review). Voisin* carefully examined, using the Green's function formalism, how three basic effects, namely the finite extent of the source, its unsteadiness (due to switch on at a given time) and the viscosity of the fluid, come into play in the build-up of the internal wave field, so that the solution calculated by Thomas & Stevenson (1972) is eventually reached.

This mode of emission is different than the one due to the translating motion of an object at constant velocity U. In the experimental work of Bonneton* & Perrier, the near field generated by an axisymmetric bell-shape hill pulled along a flat surface is studied, for a uniformly stratified fluid (Bonneton, Roux & Perrier 1995). The topographic internal waves thus generated involve non-hydrostatic effects (for instance, these waves are able to propagate downhill and to transfer most efficiently orographic effects from the surface to the upper atmosphere). This study is of direct interest for meteorology, because of the current development of non-hydrostatic weather forecast models. Two Froude numbers characterize the flow, based upon the height of the hill, h ($F = U/Nh$), and half its horizontal extent, L ($F_L = U/NL$). In the present case, $F > 0.8$, so that the fluid is able to flow over the mountain. Depending upon the value of F_L (< 1, > 1 or $\gg 1$), three regimes can be broadly distinguished. For $F_L < 1$, laminar lee waves are created, with a characteristic wavelength imposed by the horizontal extent of the hill, L. The frequency of the waves is thus imposed and, from the dispersion relation, so also is their direction of propagation: $\theta = \arccos(F_L)$; this relation is very well satisfied by the experimental measurements. For $F_L > 1$, the flow detaches from the topography and a three-dimensional recirculating zone appears behind it. Interestingly, this recirculating zone together with the topography acts as a new obstacle that generates lee waves. When $F_L \gg 1$, the hill generates a turbulent wake that emits unsteady internal waves, which supersede the lee wave and dominate the flow dynamics.

This turbulent regime is also examined experimentally and theoretically by Dupont* & Kadri. The towed object is either a sphere or an axisymmetric Gaussian hill. The basic sources of internal waves are the large-scale coherent structures of the wake, either by their collapse (for the sphere) or by their advection (for the hill). In the former case, each structure generates an impulsive wave field (Bonneton, Chomaz & Hopfinger 1993) while in the latter, a lee wave is produced (Kadri *et al.* 1996). The emitted waves then build a coherent wave field, models of which have been proposed using the linear theory of internal waves (Dupont & Voisin 1995).

Another mechanism for wave emission, related to vortex–vortex interactions at very late time ($Nt > 600$), was put forward by Spedding* & Browand in an experimental study of the wake of a towed sphere in a tank with constant N. Precise measurements were made possible by the use of a high-resolution Digital Particle Image Velocimetry technique (Fincham & Spedding 1996). To quantify the emitted wave field, Spedding* and Browand proposed an experimental adaptation of a wave–vortex decomposition derived by Riley, Metcalfe & Weissman (1981): the wave field is characterized by the divergence of the horizontal component of the velocity field.

A related problem is currently being addressed by Sutherland* & Linden, through experimental investigations of the internal wave field emitted by the coherent structures of a shear layer. In this study, the Brunt–Väisälä frequency is no longer uniform and is adjusted so that internal waves can be efficiently generated: as predicted numerically by Sutherland & Peltier (1994), N has to be small in the region of strong shear and sufficiently large in the far field.

The generation of both lee and impulsive waves found by Dupont* & Kadri and Spedding* & Browand is also encountered in a different problem, namely the collapse of a mixed region in a stably stratified fluid. Bonneton, Egermann* & Thual addressed this question numerically, in two spatial dimensions, in relation to earlier experiments performed by Wu (1969). The numerical study shows that impulsive waves are generated during the collapse of the mixed region, consistent with Wu's finding. In addition, the intrusion spreading after the collapse is found to generate waves of the lee type, while this result was not reported in Wu's experiment.

Internal waves are also excited at the point of contact with a convective zone. A mechanism of excitation by a set of isolated overshooting plumes has been proposed by Schatzman*, with application to stellar interiors (see §5). This excitation is found to be more efficient than the effect of typical random convective eddies.

The Antarctic ice shelf during winter provides a convenient outdoor laboratory (from a scientific point of view) for the study of internal waves in the stable atmospheric boundary layer (Rees & Mobbs 1988). Intense internal wave activity was detected during the STABLE experiment performed by the British Antarctic Survey (using radars and arrays of microbarographs). The analysis of these data by Rees*, Price, King & Anderson yields a detailed climatology of internal wave activity. Coherent waves have been detected, which propagate 10 m s^{-1} *faster* than the wind. Their amplitude is of order 10 Pa (100 μbar) and their wavelength is about 1 km. The physical interpretation of these results is under study.

Another outdoor laboratory where intense low-altitude internal waves are frequently observed is the northeastern part of Australia. A theory was presented by Derzho* to account for observations by Christie (1992), based upon the similarity solution of large-amplitude solitary waves with a vortex core on a deep fluid. A reasonably good agreement between this model and observations was found.

2.2. *Reflection of internal waves*

The solution of the two-dimensional Boussinesq equation for a monochromatic internal wave in a closed domain was revisited by Maas* & Lam, using concepts of dynamical systems. Their work is based on a geometric property of internal waves: when an internal wave reflects at a solid boundary, the angle of reflection is equal to the angle of incidence with respect to the *vertical* (it is set by the frequency). As discussed by Phillips (1977), the wavelength is generally reduced after one reflection, so that multiple reflections provide a *linear* mechanism of energy cascade towards small scales (this idea has later been the subject of field studies, laboratory experiments and numerical simulations – e.g. Slinn & Riley 1996).

Maas* & Lam showed that the usual cellular solutions (so called internal seiches) obtained with rectangular or elliptical domains, are atypical: with other shapes, a quite different type of standing wave is generally found, that becomes focused to an attractor (Maas & Lam 1995). The solutions are obtained geometrically by the method of characteristics, using a recursive map that determines the successive reflections of the characteristics at the boundary. Generally only a single closed characteristic is possible, and it is an attractor towards which all other characteristics converge. Moreover, dissipative and mixing effects should be localized along the attractor.

Internal waves can also be reflected in free space, when they reach a level z_c where the background Brunt–Väisälä frequency becomes smaller than the wave frequency, so that propagation becomes impossible. Sutherland* studied the influence of nonlinear effects on this problem (Sutherland 1996). The main result is that, at large amplitude, the wave packet may transmit energy across the level z_c through a nonlinear mechanism involving the generation of a mean flow interacting with the wave (Scinocca & Shepherd 1992). The transmission of energy is manifested as the generation of an internal wave packet of lower frequency. This finding has possible relevance to many geophysical applications: for instance, internal waves generated near the surface of equatorial oceans may be able to propagate to greater depth than previously believed attainable and therefore may constitute a significant momentum source for the deep equatorial countercurrent.

2.3. *Instabilities and breaking of internal waves*

In all studies reported in this section, the Brunt–Väisälä frequency N has a constant value.

The instability of a propagating plane wave was reviewed by Klostermeyer*, within the Boussinesq approximation. Results for a small-amplitude primary wave were first recalled: in a planar triad of waves, resonant interactions can be found at second order in expansions in the wave amplitude. Some of these interactions lead to wave instability, however small the primary wave amplitude. An important case is the parametric instability, exciting waves with half the primary wave frequency (and to a weaker extent harmonics of this half-frequency). This mechanism was nicely illustrated by Klostermeyer* using two coupled pendulums, one of them being tuned so that its oscillation at a given frequency triggers an oscillation of the other pendulum at half this frequency.

Klostermeyer* next presented the Floquet stability analysis, for which the perturbations are still supposed infinitesimal, but the primary wave has now a finite amplitude. The stability problem for two-dimensional perturbations was first analysed by Mied (1976) and Drazin (1977). The important conclusion of this analysis is that a propagating finite-amplitude internal wave is parametrically unstable, however large its minimum Richardson number may be. A continuous spectrum of small-scale unstable modes is obtained, plus (Klostermeyer 1990) an isolated mode. In the limit of a vanishing primary wave amplitude, the small-scale modes correspond to the resonantly interacting triads.

Klostermeyer (1991) extended this analysis to three-dimensional instabilities. When the primary wave amplitude is smaller than a threshold close to the condition for overturning, two-dimensional instability modes are found to grow fastest. Beyond this threshold, by contrast, three-dimensional instability modes dominate. One class of modes is, again, akin to a parametric instability, with frequency half the primary wave frequency. In addition, a purely growing mode, with zero frequency, is obtained:

FIGURE 1. View of noctilucent clouds from Kustavi, Finland (61°N,21°E) on 22 July 1989 showing characteristic band and streak structures. In this case, bands are separated by ~ 50 km and streaks by ~ 3 to 5 km. (From Fritts *et al.* 1993; photograph by Pekka Parviainen.)

this can be considered as a vortex mode. These modes have physical analogies with a convective instability associated with the wave overturning, but the time oscillating nature of this overturning has a significant influence.

Klostermeyer* pointed out that, in the atmosphere, two-dimensional small-scale modes have been detected by sensitive Doppler radar and the analysis of the signals has provided detailed information upon the associated large-scale primary wave. By contrast, no three-dimensional instability modes have yet been identified in radar observations. However, three-dimensional structures in noctilucent clouds, which are bright short-lived structures that form in the mesopause (figure 1), might be explained in terms of three-dimensional parametric instability.

The same Floquet problem has been solved numerically by Lombard & Riley* for different amplitudes and, especially, different angles of the wavevector of the primary wave. The purpose of this work was to interpret results of three-dimensional numerical simulations of a propagating internal wave. For small amplitudes, two-dimensional resonant interactions are found in the simulations but, ultimately, the local disturbances become three-dimensional. For an amplitude larger than that for overturning, the primary instability is complex and broad banded but is three-dimensional. These results are in agreement with Klostermeyer's analysis. An important point is that no marked difference is observed as the overturning threshold is crossed: this shows that, as the amplitude is increased, the instability goes continuously from two-dimensional resonant interactions to three-dimensional instability (possibly of the convective type) (Lombard & Riley 1996). This result strongly suggests the existence of a unique underlying mechanism, which would be of the parametric instability type according to Klostermeyer's study. Local regions of intense shear and overturning are found to eventually develop, whatever the primary wave amplitude, as a result of the instability growth. Breaking then occurs, producing three-dimensional turbulence. Very little energy is left in the main wave (about 10%), while 5% of the primary wave initial energy generates a mean flow (this point is discussed in the next section).

Bouruet-Aubertot*, Sommeria & Staquet presented the case of a standing wave confined to a square domain. The problem was investigated by two-dimensional numerical computations (in a vertical plane). The primary wave with high frequency is again unstable by parametric instability (other mechanisms occur for a low-frequency wave). The resonant interaction theory applies in this case quite similarly to the case of the propagating wave, and its prediction agrees very well with the numerical results, even for fairly large primary wave amplitude. The perturbation is observed to organize into a coherent secondary wave packet. This wave packet reaches the condition of overturning and becomes strongly unstable. This quickly leads to breaking and drives the whole flow dynamics in a kind of turbulent state; very little energy is left in the main wave (Bouruet-Aubertot, Sommeria & Staquet 1995).

These results were confirmed by laboratory experiments performed in parallel by Benielli* & Sommeria in a parallelepipedic water tank stratified with salinity (Benielli & Sommeria 1996). The wave is excited by a primary parametric instability, by vertically oscillating the tank, which induces a modulation of the apparent gravity. After its initial growth, the primary wave itself becomes unstable by parametric instability, exciting a coherent wave packet quite similar to the numerical computation. When this wave packet reaches the overturning condition, it is subjected to a rapid instability, but with convective rolls perpendicular to the main plane of the wave. These rolls are not obtained in the numerical simulation, because it is two-dimensional, but the subsequent flow behaviour is still strikingly analogous (see §4). As also found numerically by Lombard & Riley*, the experiment clearly illustrates the prediction of the Floquet theory that the wave of moderate amplitude is unstable through a two-dimensional instability, while the overturning wave undergoes a three-dimensional instability. (Note however that, strictly speaking, the Floquet theory has been derived only for the propagating wave, not the standing one.)

When rotation is present, the three-dimensional calculations of Lelong* & Dunkerton show that, by contrast, breaking of inertia–gravity waves clearly arises through Kelvin–Helmholtz instability, in agreement with observations in the middle atmosphere and with earlier theoretical predictions (Dunkerton 1984; Fritts & Rastogi 1985).

In these numerical studies, no mean flow is present initially and breaking arises through intrinsic instabilities of the wave field. The presence of a stationary horizontal flow with vertical shear provides an alternative mechanism for wave breaking, through its interaction with the propagating wave field at a critical level (where the phase speed vanishes relative to the velocity of the horizontal flow). Two numerical studies addressed this problem, presented by Andreassen* and Arendt*, Fritts & Andreassen on the one hand, and by Dörnbrack & Gerz* on the other. In both studies, breaking is found to occur through a convective instability, in agreement with Winters & d'Asaro's (1994) earlier work, which makes the problem inherently three-dimensional. A detailed study of the vortex dynamics resulting from breaking was addressed by Arendt* *et al.* (Andreassen *et al.* 1994; Fritts, Isler & Andreasson 1994). Dörnbrack & Gerz* examined the conversion of wave energy into turbulence during breaking. When a large-eddy simulation with a Smagorinsky subgrid-scale model is carried out, a $k_x^{-5/3}$ Kolmogoroff spectrum is found at the critical level during breaking (k_x being the horizontal wavenumber) (Dörnbrack, Gerz & Schumann 1995).

In the study of the lee wave regime generated by a three-dimensional topography addressed by Bonneton* & Perrier, no breaking was ever observed. When the topography is two-dimensional by contrast, waves have one degree of freedom less for dispersing so that more energetic waves propagate upwards. Consequently, wave breaking is more likely to be encountered. Peltier* studied this situation by two- and three-dimensional numerical simulations and by a Floquet stability analysis. The two-dimensional internal wave forced by the topography appears to be unstable to three-dimensional perturbations (with cross-stream wavevector), the most amplified wavelength matching the wavelength selected in the numerical simulations. Peltier* suggested that such an instability may account for the three-dimensional structure of noctilucent clouds (see figure 1), as also did Klostermeyer* who uses the same methodology but a more idealized basic flow. The same idea is shared by Fritts *et al.* (1993), who suggested that a convective instability could be at the origin of the three-dimensional structure of noctilucent clouds.

In the different stability analyses presented in this section, a normal mode approach (or modal method) is used: perturbations with an exponential time dependency are sought. As discussed by Craik*, new methods have recently been developed, for non-stratified flows, which permit one to study exactly the stability analysis of unbounded spatially simple flows, or which are employed when a Floquet theory is not available, for instance for a class of time-oscillatory three-dimensional non-axisymmetric flows. These new methods describe the stability of planar modes of arbitrarily large amplitudes, which are exact solutions of the equations of motion (e.g. a single Kelvin mode: see Craik & Criminale 1986). When Floquet theory is unavailable, Bayly, Holm & Lifschitz (1996) recently showed how to calculate growth rates; and new results were presented by Craik* (Forster & Craik 1996).

These methods have been used by Chagelishvili* to show that, within the Boussinesq approximation, a wave packet localized in a large vertical gradient of wind velocity can be subjected to an extremely rapid amplification. For uncompressible perturbations, this mechanism is very likely limited because, apart from very weak-amplitude perturbations, breaking would occur before the anomalous amplification takes place. Chagelishvili* however argued that compressible effects would come into play during the nonlinear stage of growth and could prevent breaking (Chagelishvili 1995).

3. Vortex structures in stably stratified flows

Much less is known in natural stably stratified media about small-scale vortices than about waves. Possible reasons, as argued by Etling*, are that vortices may be weaker than waves and more difficult to detect. At least two kinds of vortices have to be distinguished. The first kind corresponds to quasi-horizontal *slowly* evolving motions (compared to the waves), usually organized in layerwise structures. These are zero-frequency modes in the linear limit and, more generally, can be characterized by the fact that they contain all the potential vorticity of the flow (a quantity conserved by the inviscid dynamics). We first report on the generation of such motions by waves. A quite different kind of vortex is generated in the near wake of a body or, more generally, by strong shear or local density overturning: these are small intense vortices of characteristic time comparable to or smaller than the wave frequency (N) amidst three-dimensional turbulence. The influence of a background rotation upon the dynamics of a stably stratified flow was also addressed.

3.1. *Wave–mean (flow) interactions*

Dissipating waves can change vorticity and potential-vorticity distributions and give rise to irreversible transport of momentum from one place to another, producing significant mean-flow changes. A simple experimental demonstration was presented by McIntyre* for surface waves, using a glass oven dish filled with water and placed on an overhead projector. McIntyre* generated waves by vertically oscillating a small circular cylinder at the free surface. A streaming was then clearly driven perpendicular to the cylinder axis (this flow being visualized by chalk dust). The demonstration is described and further discussed in McIntyre & Norton (1990), who also developed a general conceptual framework for studying the mean flow generated by dissipating waves (dissipating either laminarly or by breaking). The two main ideas are (*a*) that the mean flow induced by the dissipating waves is a so-called balanced motion, that is, to good approximation is completely determined from its vorticity or potential-vorticity distribution, and (*b*) that the effective mean forces producing such mean-flow changes are just those associated with the irreversible vorticity or potential-vorticity changes caused by wave dissipation. Applications to dissipative acoustic waves, internal gravity waves and Rossby waves were presented in the paper.

McIntyre* discussed the consequences of these general processes with emphasis on the case of stratospheric Rossby waves. Rossby waves are balanced motions whose restoring mechanism depends on having a gradient of potential vorticity on isentropic (constant potential temperature) surfaces. They coexist with, indeed can often be regarded as producing, the layerwise two-dimensional turbulence of the stratosphere through a process of 'wave breaking' (e.g. McIntyre 1993). The result is a very strong spatial inhomogeneity, which is essential to understanding for instance the chemical near-isolation of the Antarctic ozone hole. The associated wave dissipation and transport of angular momentum gives rise to a persistent global-scale stratopheric 'pumping' (Holton *et al.* 1995, and references therein), air being withdrawn from the troposphere via the tropical stratosphere and pushed poleward then downward. Other atmospheric phenomena driven by wave-induced angular momentum transport include the so-called quasi-biennal oscillation (QBO), which exhibits alternating deficits and excesses of angular momentum, a half-cycle taking 14 months or so, in the equatorial stratosphere.

The final part of McIntyre's lecture was devoted to the Sun's radiative interior, below the convection zone, which, in some aspects, is fluid-dynamically similar to the Earth's stratosphere. The first question addressed was whether a QBO-like phenomenon could also exist inside the Sun, possibly contributing to observed variability and perhaps also producing material transport relevant to the observed lithium depletion (this last subject was addressed in detail in the lecture by Schatzman, see §5). The second question was how to explain the near-solid rotation indicated, by helioseismic data, for the outermost part of the Sun's interior. McIntyre* argued from potential-vorticity considerations that this would unequivocally imply the existence of an interior magnetic field strong enough to control primeval spindown (see McIntyre 1994).

Whether non-dissipative waves can also drive a balanced mean motion (that survives when the waves have gone), was discussed theoretically by Bühler* & McIntyre. Potential vorticity cannot be modified non-advectively in this case, but it was shown that advective changes in the potential-vorticity distribution can be forced by the waves. This result was illustrated by the effect of a steady gravity wave field upon a background two-dimensional potential vorticity gradient of weak amplitude (β-effect). In this case, the forcing of a balanced mean motion, in the form of a Rossby wave, arises through the wave-induced Stokes drift. A non-dissipative yet cumulative response can occur if a suitable phase-speed condition is satisfied, corresponding to resonant forcing of the Rossby wave. This problem was shown to be generalizable, for instance to three-dimensional stably stratified Boussinesq equations, by using the waves' pseudomomentum instead of the Stokes drift (Bühler 1995).

The central role played by potential vorticity also arises through the generalization of Bernoulli's theorem for steady flows to the frictional and diabatic regime: the intersection of surfaces of constant entropy and constant Bernoulli function (that relate to streamlines in the adiabatic inviscid regime) generally yields flux lines along which potential vorticity is transported (Schär 1993). A diagnostic application of this generalized Bernoulli's theorem to the numerical model output of flows around isolated topographic obstacles and to atmospheric data over the Alpes were discussed by Schär*. The net flux of potential vorticity on a surface of constant potential temperature localizes potential-vorticity anomalies and, thus, dissipation and frictional processes. Schär* showed that this net flux only depends upon the net reduction of the Bernoulli function in the wake.

An application of this work is the parametrization of the resulting drag effect in weather forecast models, as was discussed by Lott* & Miller. A simple parametrization scheme to account for the drag resulting from flows going around and over high mountains was presented. The scheme was implemented in the ECMWF forecast model and was found to have a realistic impact on the model performances when compared to the PYREX field data. Its influence on the global forecast performance of the ECMWF model was also analysed (Lott & Miller 1996).

3.2. *Vortex structures in flows without a mean shear*

Several fundamental experimental and numerical studies of vortex dynamics in unsheared flows with constant N were presented, involving either one vertical vortex, two horizontal counter-rotating vortices or a set of vortices generated by the horizontal towing of a sphere or a rake of vertical bars.

An experimental study of a single vertical columnar vortex visualized simultaneously by dye and shadowgraph was presented by Flór* & Sutherland. The vortex is generated by rotating a vertical plate over a small horizontal angle. Large vertical oscillations first appear along the vortex, due to viscous effects at its lower boundary. Nonlinear interactions between the vortex and internal wave motions next give rise to a second instability and a final layering regime associated with planar vortices is eventually reached. This study was complemented by linear stability analysis to investigate the primary instability development and its interaction with the vortex as it amplifies.

Two horizontal counter-rotating vortices are also subject to an instability, known as the Crow instability (Crow 1970). Knowledge about this instability is especially needed for civil aviation, as stressed by Robins* & Delisi (see §6). Such counter-rotating vortices develop in the wake of planes as soon as they take off, since vorticity is always generated from a three-dimensional lifting wing. Crow (1970) showed that this instability occurs in unstratified fluids. However, it most often arises in the atmosphere and the influence of a stable buoyancy was investigated by Robins & Delisi, using three-dimensional numerical simulations. Main results are that the instability grows faster than in the unstratified case and remains three-dimensional even for strong stratification. A two-dimensional numerical study (in a vertical plane) of two counter-rotating vortices was presented by Garten*, Fritts & Andreassen. For their choice of ambient density profile (exponentially decaying with altitude), different behaviours are obtained depending upon the direction of the pair motion (up or down) or whether a horizontal mean shear is applied. Preliminary results of a three-dimensional study were also displayed.

In the wake behind a sphere studied by Spedding* & Browand, stratification was found to promote a more compact, orderly and stable wake, that evolves through interactions of like-signed layerwise vortices of Gaussian shape. In spite of this layerwise structure, similarities with the unstratified wake were surprisingly found, regarding the evolution of mean turbulence quantities (Spedding, Browand & Fincham 1996). However, and this seems to be a general feature of strongly stratified flows, the initial condition has a persistent effect on the flow evolution.

When a rake of vertical bars is towed along a stably stratified tank, vertical vortices are generated in the wake of the bars. The layering structure that develops once the vertical scales have collapsed was the topic of two presentations. Holford* & Linden reported on velocity and density measurements in experiments at different Froude and Reynolds numbers. The formation of the layers, their energy and vertical scales were studied as a function of these two parameters. The very low Froude number regime was addressed by Fincham* & Maxworthy. In this case, the layers are made of quasi-two-dimensional vortices of so-called 'discus' shape. The very strong shear induced by their relative motions was found to govern the dissipation rate of the total energy (Fincham, Maxworthy & Spedding 1996). A mechanism was proposed to account for the thickness of the layers: this thickness is determined by the vertical distance over which the horizontal vorticity (due to shear) would diffuse in a single turnover time of the horizontal vortices.

As noted by McIntyre during the colloquium, such layerwise two-dimensional turbulence is like that encountered in the stratosphere, except that rotation changes the relation between layers in this case. How this coupling occurs is currently being studied by Flór* & Linden through laboratory experiments of stratified rotating flows. Monopolar and dipolar vortices are generated at different levels in a three-dimensional tank and the conditions leading to vortex alignment along the vertical are investigated. Laboratory experiments of stratified rotating flows were also presented by Boubnov*, the turbulent flow under study being either generated by sources and sinks located in the inner boundary of a cylindrical container, at a given horizontal level (Linden, Boubnov & Dalziel 1995), or resulting from a Taylor–Couette instability, an inner cylindrical container shearing the stably stratified fluid (Boubnov, Gledzer & Hopfinger 1995).

3.3. *Influence of a mean shear*

The evolution of flows with a homogeneous mean shear was reviewed by Van Atta*. Emphasis was put on a basic understanding of how the nature of this flow changes with variation of the dimensionless parameters. For this purpose, results of experiments by Piccirillo & Van Atta (1996) and of direct numerical simulations by Jacobitz, Sarkar & Van Atta (1995) were presented in a complementary way. Surprising results were found, for instance regarding the critical value of the Richardson number, Ri_{cr}, for which the turbulent kinetic energy neither grows nor decays. Ri_{cr} was found to vary substantially with two dimensionless parameters that govern the flow, the shear number SK/ϵ (the ratio of the characteristic dissipation time to the characteristic shear time) and the turbulence Reynolds number. Here $S = \mathrm{d}U/\mathrm{d}z$ is the mean vertical shear, K is the turbulent kinetic energy, and ϵ is the rate of dissipation of K. For fixed initial Reynolds number, as SK/ϵ increases from zero to large values, Ri_{cr} increases from small values of order 0.04 to maximum values of order 0.2. Beyond the maximum in Ri_{cr}, for increasingly larger values of SK/ϵ, Ri_{cr} decreases to small values. The turbulent kinetic energy also decays for large SK/ϵ. This behaviour persists even for the highest Reynolds numbers achieved. The rapid distortion analysis of Hunt, Stretch & Britter (1988) produces behaviour similar to the DNS results for large values of SK/ϵ. A simple theoretical scaling formulation accounts for the observed behaviour of Ri_{cr} over the full ranges of values of SK/ϵ and Reynolds number. This formulation was inspired by the physical insight provided by Townsend (1957).

The influence of an inhomogeneous shear upon a stably stratified fluid dramatically changes the flow dynamics and this subject was addressed through four numerical studies of a shear layer. In this flow, the nature of the primary instability already strongly differs depending upon the ratio of the thickness of the initial density and velocity profiles. When this ratio is about one, the well-known Kelvin–Helmholtz instability develops, now modified by the stable stratification; when this ratio is greater than 2.4, and the initial Richardson number at the interface, usually denoted J, is sufficiently large, the Holmboe instability develops ($J > 0.046$ for the case studied by Holmboe 1962). The non-symmetric Holmboe instability, resulting from a vertical shift of the velocity and density interfaces, which is the most frequent configuration encountered in laboratory and natural shear flows, was presented by Haigh* & Lawrence, through two-dimensional nonlinear numerical simulations.

The Kelvin–Helmholtz flow was addressed in the three other communications, attention being focused upon the role of small-scale secondary instabilities in the transition to turbulence. The value of J being low in all studies (equal to 0.05), three well-identified mechanisms are known that lead to secondary vortex structures, either in the regions in between the primary Kelvin–Helmholtz vortices, usually called braids, or in the vortex cores. In the braids, an intrinsically three-dimensional inertial mechanism leads to amplification of streamwise vorticity through vortex stretching (the stratification having an overall stabilizing effect) and yields vortex 'ribs' of alternate sign that extend along the braid. In that same braid however, the combined effect of the large-scale strain and of the baroclinic torque (due to buoyancy) results in the formation of a thin vorticity layer with strong streamwise density gradients. This layer may become unstable through a secondary Kelvin–Helmholtz instability (Corcos & Sherman 1976; Staquet 1995). As for the vortex core, the entrainment of fluid during the primary instability yields regions of heavy fluid over light fluid, an unstable situation that can bear small-scale convective rolls with axes aligned with the mean velocity (it is that same instability that leads to breaking of a wave packet at a critical level).

Fritts*, Palmer & Andreassen and Caulfield* & Peltier reported on the formation of the convective instability in the core using three-dimensional numerical simulations. Fritts* *et al.* found that streamwise vorticity is first generated in the braid, through vortex stretching, but progressively disappears (in agreement with laboratory experiments of Schowalter, Van Atta & Lasheras 1994). Streamwise vorticity next amplifies in the vortex core, through convective instability, and makes the whole structure collapse. Fritts *et al.** also presented a detailed comparison with the two-dimensional counterpart flow and with the breaking wave at a critical level (see §2). The numerical study of Caulfield* & Peltier was complemented by a linear stability analysis to spanwise perturbations of a basic flow composed of one Kelvin–Helmholtz billow; the most-amplified wavelength predicted theoretically was found to match the wavelength of the billows found numerically (Caulfield & Peltier 1994). Palmer*, Fritts & Andreassen showed that the growth of a secondary Kelvin–Helmholtz instability in the braid promotes the collapse of the flow and subsequent restratification process; however, the possible competition of this instability with the vortex stretching mechanism leading to streamwise vorticity mentioned above, which also arises in the braid, remains to be elucidated.

Finally, the stability to three-dimensional perturbations of a sheared horizontal interface in a stably and unstably stratified fluid under a general rotation field was addressed analytically and numerically by Davalos-Orozco* (Davalos-Orozco 1996). For horizontal rotation alone, he found that the momentum density difference across the interface may change the stability in some or all directions of propagation of the perturbation.

4. Statistical properties and models of stratified turbulence

While the large (planetary)-scale fluctuating motions of the Earth's stratosphere behave as two-dimensional layerwise turbulence and Rossby waves, the small scales of the stratosphere display totally different dynamics. Small scales refer here to vertical wavelengths ranging between 1 m (the experimental noise level) and 100 m. These dynamics were analysed in detail by Sidi*, from high-resolution (20 cm) measurements by fast-response thermometers and anemometers hanging below rising or descending stratospheric balloons. Two families of fluctuations are characterized from the measurements, either of the 'classical' turbulence type (for vertical scales lower than the Lumley–Shur–Ozmidov scale, for which $Fr \simeq 1$, i.e. about 20 m), or of the wave turbulence (or 'buoyancy subrange' turbulence) type. The former motions are characterized by high spatial and temporal intermittency and a quasi-isotropic Kolmogoroff ($k^{-5/3}$) spectral law for the velocity and temperature (or buoyancy) variances, thus indicating active mixing. By contrast, motions in the buoyancy subrange are quasi-permanent and quasi-homogeneous, with strongly anisotropic behaviour. Only potential energy spectra could be measured in this range of scales and were found to generally behave as k_z^{-3}, k_z being the vertical wavenumber. Their level depending upon N, these spectra can thus be written as $\alpha\ N^2\ k_z^{-3}$. However, from mid-latitude observations, Sidi* noticed a strong variability for the α coefficient, especially between the lower stratosphere or upper troposphere ($\alpha \simeq 0.15$) and the middle stratosphere ($\alpha \simeq 0.05$, above 18 km height). Also, the buoyancy flux is found to play a very weak dynamical role for this range of scales.

The importance of the latter result stems from a quasi-antinomy between two dynamical models earlier proposed to explain the -3 spectral slope found both in the stratosphere (see e.g. Sidi & Dalaudier 1989 for a review) and from fine-scale oceanic measurements (e.g. Gregg 1987). Along the -3 slope spectral range, these models predict either a strong downgradient buoyancy flux, as compared with potential and kinetic energy (Lumley 1964; Shur 1962) or, oppositely, a very weak one, even a possibly counter-gradient (restratifying) buoyancy flux (Holloway 1988). The analysis of Sidi* seems to favour the second model, and so also do the numerical simulations of stratified turbulence from breaking internal waves of Bouruet-Aubertot* *et al.*: a small and negative buoyancy flux correlation is found for different numerical experiments (Bouruet-Aubertot, Sommeria & Staquet 1996). It should be noticed that, in this work and in the corresponding experimental work by Benielli* & Sommeria, both potential energy density spectra adjust to the same $\alpha\ N^2\ k_z^{-3}$ spectra, with the same value of the constant α ($\simeq 0.2$) in the numerical and laboratory experiments.

An heuristic model of the three-dimensional spectrum of temperature fluctuations was proposed by Gurvich* (Gurvich 1995). Local axisymmetry (about the axis of gravity) is assumed and the three-dimensional energy spectrum is supposed to take constant values on ellipsoids in wavenumber space (corresponding to flattened motions in physical space). The shape of the ellipsoid is allowed to vary in wavenumber space and analytical expressions for the three-dimensional energy spectrum can thus be derived that depend upon a function characterizing this variable anisotropy. Suitable choice of this function leads to a very good fit with recent measurements of one-dimensional horizontal spectra (in $k_x^{-1.9}$) and vertical spectra ($k_z^{-3.1}$) in the stratosphere. The interest in the model is that, conversely, a three-dimensional spectrum can be derived from one-dimensional spectral measurements. An anisotropy function can then be constructed, from which the variability of the anisotropy of the medium can be estimated, an essential piece of information when the propagation of waves (such as radio waves) is to be considered in the medium.

The nonlinear dynamics of internal waves were also addressed by Winters* & d'Asaro, in the oceanic context. The three-dimensional Boussinesq equations are solved numerically and, at initial time, energy is injected at large scales through realizations of the Garrett & Munk (1979) spectrum of different amplitudes. (The Garrett & Munk spectrum is a fit of *in situ* measurements by a semi-empirical model of linear internal waves.) By contrast with the buoyancy subrange of the atmosphere, the wave interactions are weak (in the range of scales considered) and the effect of the Coriolis force is important. The determination of energy transfer rates from large to small scales and dissipation rates are the primary issues of this study. Dissipation rates are found to be in good agreement with estimations for the ocean (Gregg 1989) and range between the predictions by two theoretical models, either assuming weak interactions (McComas & Muller 1981) or using an eikonal (or ray) approach (Henyey, Wright & Flatté 1986).

The weak interaction theory has been revisited by Daubner & Zeitlin*, who applied to internal waves the formalism of weak turbulence theory successfully tested for a variety of wave phenomena (see Zakharov, L'vov & Falkovich 1992 for a review). This formalism was previously applied to internal waves with Lagrangian variables (it was for instance applied to the oceanic case by McComas & Muller 1981). The new contribution of Daubner & Zeitlin* is to use Eulerian variables, which requires a specific adaptation of the method, but is more appropriate for most applications. Dynamical equations for the Eulerian wave spectra are thus derived (restricted to the vertical plane at this stage). Two kinds of stationary non-equilibrium spectra are obtained, involving a distinct power law in the horizontal and vertical wavenumbers.

Hanazaki & Hunt* applied the rapid distortion theory (RDT) to describe the dynamics of an unsheared stably stratified homogenous flow (Hanazaki & Hunt 1996). This theory deals with the three-dimensional unsteady linear Boussinesq equations expressed in Fourier space and permits one to derive the exact temporal behaviour of any quantity from the initial conditions. Surprising behaviour is found such as, for instance, a significant buoyancy flux against the gradient (i.e. the flow restratifies) at smallest scales, for a value of the Prandtl number larger than 1; this is in very good agreement with previous laboratory experiments of Yoon & Warhaft (1990) for the same flow parameters. This suggests that the dynamics of the small scales, though very likely nonlinear, are controlled by the large energetic scales, which are linear, thus extending the validity of the RDT approach. (Note that other arguments had previously been proposed to account for this counter-gradient buoyancy flux, related to the occurrence of small-scale instabilities, e.g. Holloway 1988; Bouruet-Aubertot *et al.* 1996.)

The RDT results are able to reproduce features found in numerical and laboratory experiments initiated by grid turbulence, or by a reservoir of potential energy only (Gerz & Yamazaki 1993), very likely because linear effects have a significant dynamical role in these situations. In other situations, however, nonlinear effects must dominate the flow dynamics, for instance when the wave kinetic energy and the potential energy are initially equal at each scale. Such a configuration was chosen by Godeferd*, Staquet & Cambon to test against high-resolution (256^3) direct numerical simulations the validity of an EDQNM (Eddy Damped Quasi Normal Markovian) model adapted by Cambon (1989) to axisymmetric homogeneous unsheared stably stratified flow. The EDQNM model uses a statistical closure of nonlinear interactions and its linear version is the RDT model. A very good agreement was found between EDQNM predictions and numerical experiments when the decay of the total energy, the averaged buoyancy flux or the correlation lengths are compared. For the Reynolds numbers considered, both the EDQNM model and numerical simulations predict an anisotropic behaviour of the flow down to its smallest scales, a result that would need to be examined by the EDQNM model for higher values of the Reynolds number since it is of potential importance for small-scale modelling of geophysical flows (Godeferd, Staquet & Cambon 1995).

A new approach based on renormalization group theory has been proposed by Canuto* & Dubovikov and applied to various turbulence problems including thermal convection (Canuto & Dubovikov 1996). The case of stably stratified flows is under current investigation.

5. Mixing

5.1. *General considerations*

Understanding and predicting the mixing of chemical concentration or temperature is the final goal for many studies of internal waves or stratified turbulence. The vertical mixing is strongly restricted by the stable stratification, as it corresponds to an irreversible lifting of heavy fluid particles and lowering of light ones. This requires energy, provided at the expense of the kinetic and potential energy of the fluctuations. The rate of increase of the potential energy of the background density profile is therefore a proportion $\eta < 1$ of the rate of energy dissipation by diffusive effects for the fluctuations. Much work is devoted to the characterization of this mixing efficiency η in various conditions, as reviewed by Redondo* (see also Fernando 1991).

Dalaudier* stressed that the distinction between the background profile and fluctuations is not straightforward. A definition is provided by the sorting method of Thorpe (1977): all the fluid particles in the instantaneous density field are re-ordered according to their density, reaching the state of minimum potential energy for purely adiabatic rearrangements of the system. This is the background state, and the r.m.s. vertical displacement for this re-ordering is Thorpe's scale. Winters pointed out that this sorting method can be given a precise mathematical formulation, when it is performed in the three-dimensional domain (Winters *et al.* 1995). In practice, the density is however only measured along a vertical line, so that the sorting is only possible along this line, and the interpretation of the outcome is not as clear. Dalaudier* has extended Thorpe's sorting method to atmospheric density profiles, re-ordering fluid particles with adiabatic transforms. Density overturnings are then eliminated from the signal, which mostly modifies the Kolmogoroff turbulent range. A k^{-3} spectrum is thus obtained in the Kolmogoroff subrange after this re-ordering, extending the k^{-3} spectrum of the buoyancy subrange to smaller scales, but the physical meaning of this result is still unclear.

5.2. *Mixing by linear waves and stellar evolution*

Schatzman* proposed that random internal waves produce some mixing (i.e. transport) of chemical species in stably stratified stellar interiors (Schatzman 1993). These waves are supposedly emitted by convective plumes at the interface with the outer convective envelope. Schatzman* introduced a model predicting the spectrum of the emitted waves, their propagation and damping. The physical data are taken from a standard model of the stellar interior. The wave regime is purely linear, which is justified by estimations of the wave amplitude. A fast horizontal mixing is found ($\sim 10^3$ years). The vertical transport is much slower, and is only possible when waves are damped: otherwise fluid particles are always pulled back toward their equilibrium altitude by the buoyancy force. The damping is provided by the considerable heat diffusion. (Indeed radiative transfers proceed through multiple scattering, hence being equivalent to a strong heat diffusion.)

The model was applied to lithium, and it provides a good interpretation for the abundance of this element observed in the atmosphere of stars with different ages and mass. Lithium was supposedly produced in the primordial universe, and only burnt in the stellar core. The depletion of lithium in comparison with its primitive abundance is then due to its transport through the stably stratified interior. A good interpretation of this transport allows one to draw firm conclusions on the primitive abundance, a key test for cosmological models. Mixing by internal waves in the solar core may also partly explain the observed anomaly for the flux of solar neutrinos.

5.3. *Mixing by internal wave breaking*

In usual geophysical situations, diffusive effects are negligible on the primary waves, but gain importance after a process of inertial energy cascade toward small scales, during wave breaking events. In the case of the atmosphere, internal waves propagate upward and break due to the rarefaction of the atmosphere. By contrast in the ocean thermocline, the waves are trapped in the vertical direction. A cascade of nonlinear wave interactions then controls the dissipation of energy (see the numerical computations of Winters* & d'Asaro in §4). Once the local rate of energy dissipation is known, as well as the mixing efficiency, the rate of mixing is deduced as the product of these two quantities. The mixing efficiency is often estimated as about 0.2 in the ocean thermocline, and recent atmospheric measurements reviewed by Sidi* indicate similar values, in the range 0.15–0.3. There is a need for better understanding and for more extensive measurements of this efficiency. However the rate of mixing is mostly determined by the energy dissipation, which can vary to a large extent.

5.4. *Mixing by turbulence produced in a well-mixed region*

A standard configuration, reported by Redondo*, is an oscillating grid generating turbulence above a stably stratified interface (Redondo, Sanchez & Cantalapiedra 1996). The interface is progressively transported downward by mixing and entrainment. One of the challenging problems is to predict the velocity of this mean interface displacement, called the entrainment velocity. It is directly related to the rate of potential energy brought by turbulence. This rate depends only on the mixing efficiency, as the turbulent energy is provided in this case at a given rate by the grid motion in the well-mixed region. Unlike in the previously discussed case of wave breaking, the mixing efficiency widely depends on Ri (for further comparisons, this number is used here instead of the Froude number $Fr = Ri^{-1/2}$, although there is no shear); Ri is here a local Richardson number, based on an integral length and a r.m.s. turbulent velocity. Redondo* indicated that the mixing efficiency tends to zero for very small and very large Ri, with a maximum of 0.3 for Ri of order unity. The qualitative explanation of this behaviour is clear: for high Ri, the interface is hardly perturbed, and the turbulent energy is mostly dissipated by viscosity in the well-mixed region. For low Ri, the entrainment rate corresponds to the case of a passive tracer, and is independent of the stratification. This entrainment requires less and less energy as the stratification becomes weaker, so the mixing efficiency is reduced. The optimal transfer from the turbulent kinetic energy to potential energy occurs at intermediate *Ri*. The quantitative behaviour is however quite complex, and depends at least on a second non-dimensional parameter, for instance the ratio between the mesh of the grid and its distance to the interface. It must also be taken into account that the interface remains sharp at high *Ri*, while it becomes diffuse at low *Ri*.

General empirical laws for the mixing efficiency have been proposed, but Redondo* stresses that the analysis must concentrate on different mixing mechanisms and their efficiency before attempting a statistical description. At high *Ri*, mixing occurs very intermittently by breaking of interfacial waves. At low *Ri*, different mixing events have been analysed: vortex dipole impingment on the interface and local shear instability, involving Kelvin–Helmholtz roll-up or Holmboe-driven ejection.

5.5. *Mixing in stratified shear flows*

Mixing is often associated with velocity shear at a density interface. A thin shear layer will usually undergo shear instability (when $Ri < 1/4$) and will strongly mix until it is stabilized by stratification, with a Richardson number larger than 1/4. Then mixing proceeds at a much slower rate, by diffusive effects and sporadic breaking of internal waves. Details depend however on the flow configuration.

A careful experimental analysis of this problem has been reported by Strang* & Fernando. These experiments are performed in an Odell–Kovasznay recirculating water loop, wherein a turbulent mixed layer is driven over the stratified interface by a disk pump, which can produce various shear profiles. Mixing again occurs by shear instability for small *Ri*, and by intermittent breaking of interfacial waves at high *Ri*. These two regimes correspond to different laws for the entrainment velocity, in Ri^{-2} and Ri^{-1} respectively. Detailed measurements of energy budget, shear production and dissipation have been performed by hot-film anemometry. Laser-induced fluorescence and laser anemometry were also used. The required optical homogeneity was obtained in spite of the stratification by using an appropriate mixing of salt and alcohol solutions.

A natural example of a free stratified shear layer was presented by Walker*, Johnston, Hamill & Curran: a freshwater flow over a stationary salt wedge, remaining behind a semi-submerged barrage in the River Lagan in Belfast. The slope of the interface depends on the shear stress. This link is obtained by assuming a balance between the shear stress and pressure gradient in the horizontal direction, and hydrostatic pressure along the vertical. This shear stress has been measured for river conditions ranging from low to flood flows, and also in a laboratory model. It depends only on the parameter Re/Ri, with a law fitted by an empirical three-layer model.

A stratified shear layer also occurs in a pool-type fast breader nuclear reactor, as indicated by Cortesi*. The fluid is then liquid sodium, with low Prandtl number. The influence of this parameter has been investigated by Cortesi*, by high-resolution numerical computations, using a pseudo-spectral code. The rate of entrainment and mixing have been computed, and related to the development of the Kelvin–Helmholz billows, and merging and onset of three-dimensional structures (Cortesi 1995).

5.6. *Stratified boundary layer*

The stratified boundary layer is another configuration of great geophysical significance. Mixing near boundaries is believed to be important for the overall diapycnal mixing in the oceans. The dominance of turbulent mixing near the boundaries has been checked by Gloor*, Schürter & Wüest in the case of a small Swiss lake, stably stratified by temperature. An astonishingly regular seiching motion, with period of about one day and maximum bottom velocity 8 cm s^{-1}, is forced by the wind stress and dissipates mostly along the sloping bottom. The lake has been intensively instrumented, with velocity and temperature measurements, as well as dye injection.

The atmospheric boundary layer becomes stably stratified due to radiative cooling at the ground. This situation has been analysed with the Met Office large-eddy simulation model (Brown, Derbyshire & Mason 1994), and further studied by Maguire*, Rees & Derbyshire. This study indicates that the effect of even a shallow slope of the terrain is quite significant, especially when the geostrophic wind (high above the boundary layer) is directed along a line of constant height. Then the transverse Ekman flow driven by the Coriolis force in the boundary layer is either upward or downward. In the first case the boundary layer is found to be thinner and more strongly stratified than over a flat terrain, and it can transfer less heat flux from the atmosphere to the Earth. The opposite behaviour is obtained with a downward Ekman flow.

5.7. *Gravity currents and flows with free interfaces*

A gravity current flows along a solid boundary and creates a sheared stratified interface at its upper level. A natural example has been presented by Roget*, Imberger & Ishikawa, in a channel connecting the Japanese lake Ogawara to the sea. Salt water intrusions are forced by the tide, producing a gravity current at the channel bottom, with a quasi-stationary velocity of 35 cm s^{-1} and height 2.5 m (far from the front). This flow is limited upward by a stratified interface, with a stabilized shear ($Ri = 0.3$), and slow vertical diffusion (the eddy diffusivity is estimated as 4×10^{-6} m^2 s^{-1}). Turbulent energy is mostly confined near the bottom boundary layer, with dissipation rate $\epsilon = 10^{-5} m^2 s^{-3}$.

Gravity currents also involve unsteady deformation of the interface, especially near the front. This problem has been studied in laboratory experiments of lock release: the fluid is initially at rest, with saline water separated from fresh water by a vertical wall; this wall is suddenly released, initiating the gravity current. Petersen* has presented a numerical model adapted to this problem. A crude turbulence model is used, with a mixing length depending on a local Richardson number. Yet the initial stage of a lock release experiment by Hacker, Linden & Dalziel (1994) is well reproduced. A simplified analytical model of the same flow problem has been proposed by Moodie* and compared with numerical simulations (He & Moodie 1991).

Free interfaces are also involved in various problems of flow intrusion. Kudine*, Abramyan & Lozhkin have experimentally studied the horizontal laminar spreading of an intrusion issuing from a small volume of mixed fluid injected in a stratified medium (Abramyan & Kudine 1983). A technique of holographic interferometry provides very precise measurements of the isopycnal slope (10^{-6}). The spreading of a single intrusion is found to be in excellent agreement with an asymptotic theory by Barenblatt (1978). Complex interaction processes between two intrusions have been observed experimentally, and are not explained by current theoretical models.

Larson & Jönsson* proposed an empirical model for the dynamics of a jet vertically injected from the top of a stratified fluid. A motivation of this study is the artificial oxygenation of ponds used for fish farming. The jet is stopped by buoyancy and returns as a rising plume, as it is lighter than the surrounding fluid. The jet then interacts with this plume. The resulting maximum penetration has been calculated by Larson & Jönsson* and found in agreement with laboratory results (Larson & Jönsson 1994).

Finally complex mixing processes were analysed by Kocsis*, Gloor, Schurter &

Wüest in Lake Baikal. The stratification is very weak, since the temperature is close to 4°C, and the expansion coefficient of water nearly vanishes. This situation favours a good mixing of oxygen and nutriments, contributing to the outstanding biological activity. Measurements of pollutant residence time by Weiss, Carmack & Korpalov (1991) have indicated that cold surface water must episodically penetrate down to the bottom. Large-eddy diffusivities (10^{-3}–10^{-2} m^2 s^{-1}) were measured by Kocsis* *et al.* at the thermal front in spring, occurring at the contact point of different water masses, and contributing to the global mixing. These eddy diffusivities are at least 100 times the typical values in the open ocean thermocline (10^{-5} m^2 s^{-1}).

6. Conclusions

The subject of the conference was very wide, first because of the quite different dynamical regimes occurring in stably stratified flow. These regimes are often coupled in the same flow problem, involving for instance wave propagation and breaking, vortex dynamics, free shear flows and turbulent boundary layers. The practical context is also quite varied, with laboratory experiments, engineering problems, dynamics of lakes or ponds, oceanography, atmospheric sciences, and even astrophysics.

In geophysics, internal waves often control small-scale structures, and influence larger scales by transporting momentum, heat, salinity or various tracers. McIntyre* discussed the crucial role of such momentum transfers (by Rossby waves) in stratospheric dynamics. Internal waves are often the only source of diapycnal transport (across the isodensity surfaces). Estimating the transport of heat and carbon dioxyde by internal waves between the atmosphere and ocean interior is important for climatology. The transport of oxygen and nutriments controls the biological activity, especially in lakes and ponds. In a quite different domain, Schatzman* showed that slow lithium transport by internal waves explains observed lithium depletion in stars, and could have a significant influence on stellar evolution.

Stratified turbulence and internal waves are also encountered in engineering problems. The artificial oxygenation of fishing ponds motivated the study of Larson & Jönsson*. The influence of a stable stratification on aircraft trailing vortices was discussed by Robins* & Delisi. Such vortices can be a hazard for other aeroplanes, and their presence is one of the major limitations of airport capacity. They also influence the transport of the emitted pollutants to the stratosphere. Cortesi* stressed the influence of a thermally stratified shear layer in a sodium pool of a fast breader nuclear reactor. Various problems in chemical engineering were presented by Zimmerman*. For instance, the inhibition of heat transfer by stratification in big containers of liquefied gas can cause explosions by sudden release of a gas bubble.

However, common dynamical aspects are encountered in these various fields and this meeting was a rare opportunity to gather together researchers from these different communities and compare their results. Impressive progress in field measurements has been reported, due to very complete instrumentation, and appropriate choice of sites, providing outstanding natural laboratories. Progress in laboratory techniques has also been made, involving the appearance of spatial measurement techniques such as particle tracking, particle image velocimetry or pattern recognition. This, together with the development of direct numerical simulations, are also sources of improved analysis of the dynamics. There is then an urgent need to get more synthetic views of these flow phenomena, develop a common language and more general formulations, leading to theoretical progress.

Theoretical approaches in this field are generally adaptations of general methods of fluid dynamics or physics. Interestingly, even classical ideas often led to new and unexpected results when adapted to the case of stratified fluids. For instance new results for linear internal waves were reported, concerning wave generation and properties of standing waves in a closed basin. The statistical description in the classical weakly nonlinear regime has been also revisited. Rapid distortion theory provides subtle results, like countergradient heat flux, which is surprising for a linear theory. This provides interesting perspectives for the modelling of complex, unsteady, flow problems involving stratification. Beyond this linear analysis, two-point closure models, developed in the sixties for isotropic turbulence, provide new insight in the strongly turbulent regime, when anisotropy is introduced.

Progress has been made in the identification of various instabilities and resulting flow structures occurring in wave breaking. However, as stressed by a few speakers, most stably stratified flows have distinctive characteristics dependent on the initial and boundary conditions (see e.g. Hunt & Carruthers 1990). Introducing this deterministic aspect into a statistical description of wave fields or turbulence remains a great unsolved problem. Global flow models have been improved and compared to laboratory experiments or observations of natural phenomena, using empirical mixing length models for turbulence. There is a clear need to continue to develop such global models to interpret and complement laboratory or field measurements, and to incorporate the new theoretical developments into practical modelling tools. In particular, there is a wide gap between our physical insight into mixing processes, and the very crude parametrization still used in models of general circulation for the atmosphere or the oceans. Attempts have been made to fill this gap, but much progress remains to be done.

We thank all participants for helpful comments on a preliminary version of the manuscript. C.S. is grateful to Professor E.J. Hopfinger for his help in organizing the colloquium.

Financial support was provided by Association Universitaire de Mécanique; Centre National de la Recherche Scientifique (SPI Dept); Centre Militaire d'Océanographie (EPSHOM); Electricité de France; the European Office of the US Navy; GDR Mécanique Fondamentale des Fluides Géophysiques et Astrophysiques; IFREMER; Ministère de l'Education Nationale, de l'Enseignement Supérieur, de la Recherche et de l'Insertion Professionnelle.

REFERENCES

Abramyan, T. O. & Kudine, A. M. 1983 A laboratory study of the interaction of patches of mixed liquid spreading in a stratified medium. *Izv. Atmos. Ocean Phys.* **19**, 670–673.

Andreassen, O., Wasberg, C. E., Fritts, D. C. & Isler, J. R. 1994 Gravity wave breaking in two and three dimensions. 1. Model description and comparison of two-dimensional evolutions. *J. Geophys. Res.* **99** D4, 8095–8108.

Barenblatt, G. I. 1978 Dynamics of turbulent spots and intrusions in a stably stratified fluid. *Izv. Atmos. Ocean. Phys.* **14**, 139–145.

Bayly, B., Holm, D. & Lifschitz, A. 1996 Three-dimensional stability of elliptical vortex columns in external strain flows. *Phil. Trans. R. Soc. Lond.* (to appear).

Benielli, D. & Sommeria, J. 1996 Excitation of internal waves by parametric instability and stratified turbulence. *Dyn. Atmos. Oceans* **23**, 335–343.

Bonneton, P., Chomaz, J.-M. & Hopfinger, E. J. 1993 Internal waves produced by the turbulent wake of a sphere moving horizontally in a stratified fluid. *J. Fluid Mech.* **254**, 23–40.

Bonneton, P., Roux, N. & Perrier, M. 1995 Etude expérimentale de la dynamique des ondes de relief. *Notes du Centre National de Recherches Météorologiques* 12, Mai 1995.

BOUBNOV, B. M., GLEDZER, E. B. & HOPFINGER, E. J. 1995 Stratified circular Couette flow: instability and flow regimes. *J. Fluid Mech.* **292**, 333–358.

BOURUET-AUBERTOT, P., SOMMERIA, J. & STAQUET, C. 1995 Breaking of standing internal gravity waves through two-dimensional instabilities. *J. Fluid Mech.* **285**, 265–301.

BOURUET-AUBERTOT, P., SOMMERIA, J. & STAQUET, C. 1996 Stratified turbulence produced by internal wave breaking: two-dimensional numerical experiments. *Dyn. Atmos. Oceans* **23**, 371–378.

BROWN, A. R., DERBYSHIRE, S. H. & MASON, P. J. 1994 Large eddy simulation of stable atmospheric boundary layers with a revised stochastic subgrid scale model. *Q. J. R. Met. Soc.* **120**, 1485–1512.

BÜHLER, O. 1995 Waves and balanced mean flows in the atmosphere. PhD thesis, Cambridge University.

CAMBON, C. 1989 Spectral approach to turbulence in a stratified fluid. In *Advances in Turbulence 2* (ed. H.-H. Fernholz & H. E. Fiedler). Springer.

CANUTO, V. M. & DUBOVIKOV, M. S. 1996 A dynamical model for turbulence: I. General formalism. *Phys. Fluids* **8**, 571–586.

CAULFIELD, C. P. & PELTIER, W. R. 1994 Three-dimensionalization of the stratified mixing layer. *Phys. Fluids* **6**, 3803–3805.

CHAGELISHVILI, G. D. 1995 Onto peculiar evolution of internal gravity waves in high shear winds. *J. Fluid Mech* (submitted).

CHRISTIE, D. R. 1992 The morning glory of the Gulf of Carpentaria: a paradigm for nonlinear waves in low atmosphere. *Austral. Met. Mag.* **41**, 21–60.

CORTESI, A. B. 1995 Direct numerical simulations of the temporally developing, stably-stratified mixing layer for a wide range of Richardson and Prandtl numbers. PhD dissertation, ETH Zurich.

CRAIK, A. D. D. & CRIMINALE, W. O. 1986 Evolution of wavelike disturbances in shear flows: a class of exact solutions of the Navier-Stokes equations. *Proc. R. Soc. Lond.* A **406**, 13–26.

CROW, S. C. 1970 Stability theory for a pair of trailing vortices. *AIAA J.* **8**, 2172–2179.

DAUBNER, S. & ZEITLIN V. 1996 On the stationary energy spectra for unidirectionally propagating internal waves. *Phys. Lett.* A (submitted).

DÁVALOS-OROZCO, L. A. 1996 Rayleigh-Taylor instability of a two-fluid system under a general rotation field. *Dyn. Atmos. Oceans* **23**, 247–255.

DÖRNBRACK, A., GERZ, T. & SCHUMANN, U. 1995 Turbulent breaking of overturning gravity waves below a critical level. *Appl. Sci. Res.* **54**, 163–176.

DRAZIN, P. G. 1977 On the instability of an internal gravity wave. *Proc. R. Soc. Lond.* A **356**, 411–432.

DUNKERTON, T. J. 1984 Inertio-gravity waves in the stratosphere. *J. Atmos. Sci.* **41**, 3396–3404.

DUPONT, P. & VOISIN, V. 1996 Internal waves generated by a translating and oscillating sphere. *Dyn. Atmos. Oceans* **23**, 289–298.

FERNANDO, H. J. S. 1991 Turbulent mixing in stratified fluids. *Ann. Rev. Fluid Mech.* **23**, 455–493.

FINCHAM, A. M., MAXWORTHY, T. & SPEDDING, G. R. 1996 Energy dissipation and vortex structure in freely-decaying stratified grid-turbulence. *Dyn. Atmos. Oceans* **23**, 155–169.

FINCHAM, A. M. & SPEDDING, G. R. 1996 Low cost, high resolution DPIV for turbulent flows. *Exps. Fluids* (submitted).

FORSTER, G. K. & CRAIK, A. D. D. 1996 The stability of three-dimensional time-periodic flows with elliptical streamline. *J. Fluid Mech.* (submitted).

FRITTS, D. C., ISLER, J. R., THOMAS, G. E. & ANDREASSEN, O. 1993 Wave breaking signatures in noctilucent clouds. *Geophys. Res. Lett.* **20**, 2039.

FRITTS, D. C., ISLER, J. R. & ANDREASSEN, O. 1994 Gravity wave breaking in two and three dimensions. 2. Three-dimensional evolution and instability structure. *J. Geophys. Res.* **99** D4, 8109–8123.

FRITTS, D. C. & RASTOGI, P. K. 1985 Convective and dynamical instabilities due to gravity wave motions in the lower and middle atmosphere: Theory and observations. *Radio Sci.* **20**, 1247–1277.

GARRETT, C. & MUNK, W. 1979 Internal waves in the ocean. *Ann. Rev. Fluid Mech.* **11**, 339–369.

GERZ, T. & YAMAZAKI, H. 1993 Direct numerical simulations of buoyancy driven turbulence in stably stratified fluid. *J. Fluid Mech.* **249**, 415–440.

GODEFERD, F. S., STAQUET, C. & CAMBON, C. 1995 Stably stratified turbulence: a statistical two-point closure vs. direct numerical simulations. *Proc. Xth Symp. on Turbulent Shear Flows.*

GREGG, M. 1987 Diapycnal mixing in the thermocline: a review. *J. Geophys. Res.* **92** C5, 5249–5286.

GREGG, M. C. 1989 Scaling turbulent dissipation in the thermocline. *J. Geophys. Res.* **94** C7, 9686–9698.

GURVICH, A. S. 1995 Model of three-dimensional spectrum of locally axisymmetric temperature inhomogeneities in stably stratified atmosphere. *Izv. Atmos. Ocean. Phys.* **31**, 211–216.

HACKER, J., LINDEN, P. F. & DALZIEL, S. B. 1994 Mixing in lock-release gravity currents. *Proc. Fourth Intl Symp. Stratified Flows, Grenoble, France*, vol. 4, §B6.

HANAZAKI, H. & HUNT, J. C. R. 1996 Linear processes in unsteady stably stratified turbulence. *J. Fluid Mech.* (to appear).

HE, Y. & MOODIE, T. B. 1991 The signalling problem in nonlinear hyperbolic wave theory. *Stud. Appl. Maths* **85**, 343–372.

HENYEY, F. S., WRIGHT, J. & FLATTÉ, S. 1986 Energy and action flow through the internal wavefield: an eikonal approach. *J. Geophys. Res.* **91**, 8487–8495.

HOLLOWAY, G. 1988 The buoyancy flux from internal gravity wave breaking. *Dyn. Atmos. Oceans* **12**, 107–125.

HOLMBOE, J. 1962 On the behavior of symmetric waves in stratified shear layers. *Geofys. Publ.* **24**, 67–113.

HOLTON, J. R., HAYNES, P. H., MCINTYRE, M. E., DOUGLASS, A. R., ROOD R. B. & PFISTER, L. 1995 Stratosphere–troposphere exchange. *Rev. Geophys.* **33**, 403–439.

HUNT, J. C. R. & CARRUTHERS, D. J. 1990 Rapid distortion theory and the 'problems' of turbulence. *J. Fluid Mech.* **212**, 497–532.

HUNT, J. C. R., STRETCH, D. D. & BRITTER, R. E. 1988 Length scales in stably stratified turbulent flows and their use in turbulence models. In *Stably Stratified Flow and Dense Gas Dispersion* (ed. J. S. Puttock), pp. 285–321. Clarendon.

JACOBITZ, F., SARKAR, S. & VAN ATTA, C. W. 1995 Numerical simulation of a stratified turbulent shear flow. *J. Fluid Mech.* (submitted).

KADRI, Y., BONNETON, P., CHOMAZ, J. M. & PERRIER, M. 1996 Stratified flow over three-dimensional topography. *Dyn. Atmos. Oceans* **23**, 321–334.

KLOSTERMEYER, J. 1990 On the role of parametric instability of internal gravity waves in atmospheric radar observations. *Radio Sci.* **25**, 983.

KLOSTERMEYER, J. 1991 Two and three-dimensional parametric instabilities in finite amplitude internal gravity waves. *Geophys. Astrophys. Fluid Dyn.* **61**, 1–25.

LARSON, M. & JÖNSSON, L. 1994 Mixing in a two-layer stably stratified fluid by a turbulent jet. *J. Hydraul. Res.* **32**, 271–289.

LIGHTHILL, J. 1978 *Waves in Fluids*. Cambridge University Press.

LINDEN, P .F., BOUBNOV, B. M. & DALZIEL, S. B. 1995 Source-sink turbulence in a rotating stratified fluid. *J. Fluid Mech.* **298**, 81–112.

LOMBARD, P. N. & RILEY, J. J. 1996 On the breakdown into turbulence of propagating internal waves. *Dyn. Atmos. Oceans* **23**, 345–355.

LOTT, F. & MILLER, M. J. 1996 A new subgrid scale orographic drag parametrization; its testing in the ECMWF model. *Q. J. R. Met. Soc* (to appear).

LUMLEY, J. L. 1964 The spectrum of nearly inertial turbulence in a stably stratified fluid. *J. Atmos. Sci.* **21**, 99–102.

MAAS, L. R. M. & LAM, F.-P. A. 1995 Geometric focusing of internal waves. *J. Fluid Mech.* **300**, 1–41.

MCCOMAS, C. H. & MULLER, P. 1981 The dynamic balance of internal waves. *J. Phys. Oceanogr.* **11**, 970–989.

MCINTYRE, M. E. 1993 On the role of wave propagation and wave breaking in atmosphere–ocean dynamics. *Proc. XVIII Intl Congr. Theor. Appl. Mech., Haifa*, pp. 281–304. Elsevier.

MCINTYRE, M. E. 1994 The quasi-biennal oscillation (QBO): some points about the terrestrial QBO and the possibility of related phenomena in the solar interior. *Proc. NATO Adv. Workshop on The Solar Engine and its Influence on the Terrestrial Atmosphere and Climate* (ed. E. Nesme-Ribes). Springer.

MCINTYRE, M. E. & NORTON, W. A. 1990 Dissipative wave-mean interactions and the transport of vorticity or potential vorticity. *J. Fluid Mech.* **212**, 403–435.

MIED, R. P. 1976 The occurence of parametric instabilities in finite amplitude internal gravity waves. *J. Fluid Mech.* **78**, 763–784.

PHILLIPS, O. M. 1977 *The Dynamics of the Upper Ocean.* Cambridge University Press.

PHILLIPS, O. M. 1981 Wave interactions – the evolution of an idea. *J. Fluid Mech.* **106**, 215–227.

PICCIRILLO, P. & VAN ATTA, C. W. 1996 The evolution of a uniformly sheared thermally stratified shear flow. *J. Fluid Mech.* (to appear).

REDONDO, J. M., SANCHEZ, M. A. & CANTALAPIEDRA, I. R. 1996 Turbulent mechanisms in stratified fluids. *Dyn. Atmos. Oceans* **24**, 107–115.

REES, J. M. & MOBBS, S. D. 1988 Studies of internal gravity waves at Halley Base, Antartica, using wind observations. *Q. J. R. Met. Soc.* **114**, 939–966.

RILEY, J. J., METCALFE, R. W. & WEISSMAN, M. A. 1981 Direct numerical simulations of homogeneous turbulence in density stratified fluids. In *Nonlinear Properties of Internal Waves, AIP Conf. Proc.* (ed. B. J. West), pp. 79–112.

SCHÄR, C. 1993 A generalization of Bernoulli's theorem. *J. Atmos. Sci.* **50**, 1437–1443.

SCHATZMAN, E. 1993 Transport of angular momentum and diffusion by the action of internal waves. *Astron. Astrophys.* **279**, 431.

SCHOWALTER, D. G, VAN ATTA, C. W. & LASHERAS, J. C. 1994 Baroclinic generation of streamwise vorticity in a stratified shear layer. *Meccanica* **20**, 361–372.

SCINOCCA, J. & SHEPHERD, T. 1992 Nonlinear wave-activity conservation laws and Hamiltonian structure for the two-dimensional anelastic equations. *J. Atmos. Sci.* **49**, 5–27.

SHUR, G. N. 1962 Experimental investigations of the energy spectrum of atmospheric turbulence. *Trudy 43*, pp. 79–90 (transl. AID. Rep. T63-55, Aerospace Information Division, Library of Congress).

SIDI, C. & DALAUDIER, C. 1989 Temperature and heat flux spectra in the turbulent buoyancy range. *Pure Appl. Geophys.* **130**, 547–569.

SLINN, D. N. & RILEY, J. J. 1996 Turbulent mixing in the oceanic boundary layer due to internal wave reflection from sloping terrain. *Dyn. Atmos. Oceans* (in press).

SPEDDING, G. R., BROWAND, F. K. & FINCHAM, A. M. 1996 The long-time evolution of the initially-turbulent wake of a sphere in a stable stratification. *Dyn. Atmos. Oceans* **23**, 171–182.

STAQUET, C. 1995 Two-dimensional secondary instabilities in a strongly stratified shear layer. *J. Fluid Mech.* **296**, 73–126.

SUTHERLAND, B. R. 1996 Internal gravity wave radiation into weakly stratified fluid. *Phys. Fluids* **8**, 430–441.

SUTHERLAND, B. R. & PELTIER, W. R. 1994 Turbulence transition and internal wave generation in density stratified jets. *Phys. Fluids* **6**, 1267–1284.

THOMAS, N. H. & STEVENSON, T. N. 1972 A similarity solution for viscous internal waves. *J. Fluid Mech.* **54**, 495–506.

THORPE, S. A. 1977 Turbulence and mixing in a Scottish loch. *Phil. Trans. R. Soc. Lond.* **286**, 125–181.

TOWNSEND, A. A. 1957 Turbulent flow in a stably stratified atmosphere. *J. Fluid Mech.* **3**, 499–523.

WEISS, R. F., CARMACK, E. C. & KORPALOV, V. M. 1991 Deep water renewal and biological production in Lake Baikal. *Nature* **349**, 665–669.

WINTERS, K. B. & D'ASARO, E. A. 1994 Three-dimensional wave breaking near a critical level. *J. Fluid Mech.* **272**, 255–284.

WINTERS, K. B., LOMBARD, P. N., RILEY, J. J. & D'ASARO, E. A. 1995 Available potential energy and mixing in density-stratified fluids. *J. Fluid Mech.* **289**, 115–128.

WU, J. 1969 Mixed region collapse with internal wave generation in a density stratified medium. *J. Fluid Mech.* **35**, 531–544.

YOON, K. & WARHAFT, Z. 1990 The evolution of grid generated turbulence under conditions of stable thermal stratification. *J. Fluid Mech* **215**, 601–638.

ZAKHAROV, V. E., L'VOV, V. S. & FALKOVICH, G. 1992 *Kolmogoroff Spectra of Turbulence. Part 1. Wave Turbulence.* Springer.

ZIMMERMAN, W. B. & VELARDE, M. G. 1994 Nonlinear waves in stably stratified dissipative media-solitary waves and turbulent bursts. *Physica Scripta* **T55**, 111–114.